K_o-OCR Relationships in Soil

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ABSTRACT: The relationships between K_o and OCR are investigated for primary loading-unloading-reloading conditions. The study reviews laboratory data from over 170 different soils and presents an approach common to clays, silts and sands. Simple empirical methods for predicting K_o for normally consolidated and overconsolidated soils are evaluated. The validity of the methods is supported by statistical analyses. On the basis of the findings, only the effective stress friction angle (ϕ') and prior stress history (OCR and OCR_{max}) are needed to predict approximate values of K_o.

Introduction

The prediction of the in-situ state of stress in soil is of major importance in a wide variety of geotechnical problems. Numerous investigators have addressed this problem and have achieved varying degrees of success. Although a substantial data base has been developed, it is still not possible to predict exactly the in-situ state of stress in most natural soil deposits because they have undergone a complex stress history of loading and unloading which is difficult to reconstruct precisely.

The geostatic vertical stress can be estimated from a profile of effective overburden stress with depth. The in-situ horizontal stress, however, is highly dependent on the geologic history of the soil. It is common to represent the ratio of horizontal to vertical effective stress by the at-rest coefficient:

$$K_o = \frac{\sigma_h'}{\sigma_v'} \qquad (1)$$

Consider the simplified stress history depicted in Fig. 1 for a homogeneous soil deposit with horizontal ground surface. Stress path OA represents virgin loading of the soil deposit, associated with sedimentation and normally-consolidated conditions. As represented by Fig. 1, the at-rest coeficient remains constant during virgin compression (K_{onc}). Any reduction in the effective overburden stress results in overconsolidation of the soil, represented by path ABC. Mechanisms causing an overconsolidated effect include erosion, excavation, rise of

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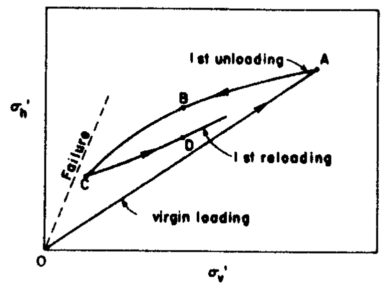

FIG 1.—Simplified Stress History of Soil under K_o Conditions

the groundwater table, removal of surcharge loads, etc. During unloading, the overconsolidation ratio, OCR = $\sigma'_{v_{max}}/\sigma'_v$, has a pronounced effect on the value of K_o. If loading is reapplied after simple rebound, the reload relationship subsequently will follow a path similar to CD in Fig. 1. Subsequent unloading and reloading, for example by seasonal water table fluctuations, is likely to cause stress paths to occur within the loop ABCDA.

To evaluate the behavior of horizontal stresses during vertical load-unload-reload conditions, available laboratory K_o data were collected from various sources published in the geotechnical literature. This study includes data compiled from over 170 different soils tested and reported by many researchers. Tables 1 and 2 contain a summary of the virgin load-unload data for cohesive and cohesionless soils, respectively, with relevant index properties.

The soils included in this study come from a wide variety of sources. Many factors exist which could not be evaluated quantitatively, including: (1) K_o test method; (2) different equipment and research personnel; (3) sampling disturbance effects; (4) time and aging effects; (5) inherent lateral anisotropy, etc., and (6) errors and differences associated with other relevant soil properties (ϕ', D_r, etc). The problems associated with laboratory K_o testing have been considered in (Refs. 5, 6, 16, 38, 66, 75, and 76). Difficulties in field measurements of K_o are described by others (37,44,45,72,73,77).

The objective of this study is to delineate the behavior of K_o during simple loading-unloading-reloading, corresponding to the virgin compression of normally-consolidated soils, subsequent rebound or swelling associated with overconsolidated soils, and recompression under conditions of no lateral yield. A wealth of data has been accumulated for simple load-unload conditions. Only a few soils reported in the literature also have been tested under reload conditions.

Normally-Consolidated Soil.—Several theoretical and empirical relationships for K_{onc} have been postulated for normally-consolidated clays and sands (6, 12,14,24,37,63). Probably the simplest and most widely known is the approximation to the theoretical formula by Jáky (28) for primary loading:

$$K_{onc} = 1 - \sin \phi' \quad \text{(2)}$$

in which ϕ' = the effective stress friction angle. Fig. 2 shows that this relationship is reasonable for the cohesive soils in Table 1. A best fit line (assumed intercept b = 1) constructed between K_{onc} and $\sin \phi'$ indicated

$$K_{onc} = 1 - 0.987 \sin \phi' \quad \text{(3)}$$

having a sample correlation coefficient r = 0.854. In other words, ϕ' accounts for 73% (or r^2) of the variability observed in K_{onc} values of normally-consolidated clays.

A similar statistical analysis conducted on K_{onc} for cohesionless soils (Fig. 2) determined

$$K_{onc} = 1 - 0.998 \sin \phi' \quad \text{(4)}$$

in which r = 0.625. The data of Sherif, et al. (62) and Al-Hussaini and Townsend (5) were weighted so as not to bias the statistical trend toward one or more researchers who contributed large amounts of data. These two sets of data each accounted for only 5% of the summation totals (Σx, Σx^2, etc.) used in calculating linear regressions, although together they comprise approximately 75% of the total data base for sands listed in Table 2.

A review of all available data for both clays and sands (total of 121 points) indicated the following best fit line from linear regression analysis (r = 0.802):

$$K_{onc} = 1 - 1.003 \sin \phi' \quad \text{(5)}$$

Numerous investigators have suggested that K_{onc} may correlate with liquid limit, plasticity index, clay fraction, uniformity coefficient, void ratio, or other index properties of the soil. The data collected during this study did not confirm any of these relationships. However, the Jáky formula (Eq. 2) was supported by these data.

Horizontal Stress During Unloading.—Overconsolidation because of rebound results in higher values of K_o than the K_{onc} values obtained during virgin compression. One of the "classic references" for an observed K_{ou}-OCR relationship was presented by Brooker and Ireland (12), although their conclusions are based on the data of only five soils. Another empirical approach was proposed by Sherif and Koch (63). Dayal, et al. (18) recommended a method of curve fitting K_{ou} data, requiring two soil parameters. Wroth (77) derived relationships for lightly to heavily overconsolidated soils. Mitachi and Kitago (47) present an analysis which requires determination of the isotropic and one-dimensional anisotropic swelling indices. Pender (54) developed a critical-state model of overconsolidated soil which predicts K_{ou} during swelling.

Alternatively, the variation of K_{ou} with OCR may be expressed simply as a function of the effective stress friction angle, ϕ', as hypothesized by Schmidt (61) and Průška (58). This approach has a distinct advantage since only one soil parameter is required for predicting both normally-consolidated and overconsolidated values of K_o, as well as defining soil strength. The simplest relationship proposed is that given by Schmidt (61) for K_{ou} during primary unloading:

$$\frac{K_{ou}}{K_{onc}} = \mathrm{OCR}^{\alpha} \quad \text{(6)}$$

in which α = an exponent defined as the at-rest rebound parameter of the soil. This approach has subsequently been used by others (7,21,37,41,45,56,60, 66,72). The compiled K_{ou}-OCR data are presented in Figs. 3 and 4 for the soils considered in this study. For clarification, the data of several soils have been

TABLE 1.—Summary of K_o Data for Clays during Virgin Load-Unload

Number (1)	Soil name (2)	Natural water content, w_n, as a percentage (3)	Liquid limit w_L, as a percentage (4)	Plasticity index, PI, as a percentage (5)	Clay content, as a percentage less than two microns (6)	Effective friction angle ϕ', in degrees (7)	Maximum OCR (8)	Earth pressure coefficient, K_{onc} (9)	Rebound exponent, α (10)	Sample correlation coefficient, r (11)	Reference (12)
1	Spestone Kaolin	—	72	32	—	22.6	2.6	0.64	0.66	0.994	Parry and Nadarajah (51)
2	Sydney Kaolin	—	50	16	—	30.7	8.0	0.48	0.47	0.971	Poulos (56)
3	Hydrite 10 Kaolinite (floc.)	—	62	28	96	17.8	17.5	0.75	0.30	0.986	Abdelhamid and Krizek (1)
4	Hydrite 10 Kaolinite (disp.)	—	62	28	96	16.9	17.5	0.69	0.25	0.975	Abdelhamid and Krizek (1)
5	Hydrite PX Kaolinite	112	—	—	—	16.9	15.4	0.65	0.19	0.990	Edil and Dhowian (19)
6	Australian Kaolin 1	—	75	40	62	23.0	—	0.56	—	—	Moore and Cole (48)
7	Australian Kaolin 2	—	58	32	40	30.0	—	0.44	—	—	Moore and Cole (48)
8	Kaolin	—	—	—	—	23.2	5.2	0.64	0.38	0.991	Parry and Wroth (52)
9	Spestone Kaolin	—	76	37	68	20.7	4.0	0.66	0.29	0.989	Sketchley and Bransby (70)
10	Kaolin	—	—	—	—	23.0	7.8	0.69	0.28	0.994	Burland (14)
11	Kaolin	—	55	23	40	23.3	40.0	0.51	0.30	0.998	Singh (66)
12	London Clay	32	95	65	52	20.0	44.0	0.65	0.46	0.959	Skempton (69)
13	London Clay	—	65	38	64	17.5	32.0	0.66	0.37	0.999	Brooker and Ireland (12)
14	London Clay	—	—	41	—	—	—	—	0.46	0.960	Som (71)
15	Weald Clay	—	41	21	39	22.0	32.0	0.54	0.49	0.995	Brooker and Ireland (12)
16	Weald Clay	—	46	24	38	25.9	7.8	0.60	0.39	0.996	Henkel and Sowa (25)
17	Weald Clay	—	—	—	—	26.2	8.6	0.58	0.33	0.992	Skempton and Sowa (69)
18	Weald Clay	—	—	—	—	—	2.5	0.68	0.40	0.996	Parry and Amerasinghe (51)
19	Bearpaw Shale	—	101	78	59	15.5	32.0	0.70	0.27	0.996	Brooker and Ireland (12)
20	Bearpaw Shale	—	82	64	50	21.0	35.7	0.65	0.41	0.992	Singh (67)
21	Drammen Clay	—	—	29	—	—	—	0.50	—	—	Prevost (57)
22	Drammen Clay 1	52	60	31	—	31.7	—	0.49	—	—	Berre and Bjerrum (8)
23	Drammen Clay 2	32	33	10	—	30.0	—	0.49	—	—	Berre and Bjerrum (8)
24	Drammen Clay	41	55	27	—	30.7	50.0	0.49	0.45	0.993	Brown (13)
25	New York Varved Clay	—	65/35	39/12	—	20.9	20.0	0.67	0.34	0.999	Leathers and Ladd (41)
26	Hackensack Valley Varved Clay	49	65/40	35/25	—	19.0	4.1	0.65	0.36	—	Saxena (60)
27	Connecticut Valley Varved Clay	—	—	28	—	—	16.0	0.67	0.40	1.000	Saxena (60)
28	South African Clay	—	—	—	—	28.7	—	0.48	0.39	0.946	Knight and Blight (32)
29	Seattle Clay	—	52	26	—	28.8	8.4	0.65	0.24	0.906	Sherif and Strazer (64)
30	Seattle Clay 2-1	27	47	18	53	—	6.0	0.61	0.43	0.996	Sherif and Koch (63)
31	Seattle Clay 3	23	38	10	—	—	14.7	0.52	0.45	0.994	Sherif and Koch (63)
32	Hokkaido Clay 1	—	52	21	—	36.2	10.7	0.45	0.53	0.975	Mitachi and Kitago (47)
33	Hokkaido Clay 2	—	52	21	—	35.0	8.5	0.45	0.44	0.906	Mitachi and Kitago (47)
34	Hokkaido Clay 3	—	72	32	—	35.1	10.7	0.47	0.52	0.981	Mitachi and Kitago (47)
35	Nebraska Clay 1	—	—	42	—	—	—	0.59	0.44	—	Geotechnical Engrs. (21)

TABLE 1.—Continued

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
36	Nebraska Clay 2	—	—	61	—	—	—	0.78	0.33	—	Geotechnical Engrs. (21)
37	Nebraska Clay 3	—	—	87	—	—	—	0.78	0.35	—	Geotechnical Engrs. (21)
38	Nebraska Clay 4	—	—	102	—	—	—	0.80	0.47	—	Geotechnical Engrs. (21)
39	Portsmouth Clay	50	35	15	—	32.0	8.0	0.47	0.46	0.998	Simon, et al. (65)
40	Beaumont Clay	26	67	41	—	24.0	5.0	0.55	0.36	0.932	Mahar and Ingram (43)
41	Boston Blue Clay	—	41	21	—	26.8	8.0	0.54	0.40	0.997	Kinner and Ladd (30)
42	Boston Blue	—	33	15	30	26.5	32.0	0.48	0.45	0.999	Ladd (35)
43	Chicago Clay	—	28	10	36	26.3	32.0	0.46	0.53	0.994	Brooker and Ireland (12)
44	Goose Lake Flour	—	32	16	31	27.5	32.0	0.50	0.49	0.994	Brooker and Ireland (12)
45	Albuquerque Clay-Sand	—	25	11	18	30.5	8.0	0.56	0.37	0.990	Calhoun and Triandafilidis (15)
46	Backebol Clay	95	90	60	—	30.0	—	0.49	—	—	Massarsch and Broms (44)
47	Bombay Clay	—	115	70	48	24.0	24.4	0.63	0.39	0.994	Kulkarni (33)
48	Portogruaro Silt	28	36	13	27	—	64.0	0.41	0.39	0.980	Bellotti, et al. (7)
49	Porto Talle Clay	32	44	21	20	—	64.0	0.53	0.41	0.990	Bellotti, et al. (7)
50	Tarquinia Silty Clay	28	43	24	39	—	64.0	0.58	0.43	0.985	Bellotti, et al. (7)
51	Tarquinia Clay	22	58	44	55	—	64.0	0.65	0.49	0.985	Bellotti, et al. (7)
52	Catania Clay	37	78	54	75	—	64.0	0.70	0.43	0.990	Bellotti, et al. (7)
53	Pisa Clay	24	57	36	44	—	64.0	0.44	0.58	0.995	Bellotti, et al. (7)
54	Chiani Clay	61	92	62	70	—	64.0	0.62	0.46	0.980	Bellotti, et al. (7)
55	Parana Clay	32	55	33	69	—	64.0	0.65	0.49	0.995	Bellotti, et al. (7)
56	Triesta Clay	47	70	48	32	—	64.0	0.55	0.52	0.995	Bellotti, et al. (7)
57	Leda Clay	—	—	24	—	—	—	—	0.38	0.950	Kelly (29)
58	Khor-Al-Zubair Clay	42	55	35	—	27.3	5.0	0.49	0.40	0.994	Hanzawa (22)
59	Fao Clay	—	39	20	—	36.9	—	0.44	—	—	Hanzawa (23)
60	Jarva Krog Clay	58	50	22	—	—	—	0.41	—	—	Massarsch, et al. (45)
61	Ska-Edeby Clay	70	55	30	—	—	—	0.52	—	—	Massarsch, et al. (45)
62	Ursvik Clay	55	45	25	—	—	—	0.47	—	—	Massarsch, et al. (45)
63	Kalix Clay	120	160	105	—	—	—	0.52	—	—	Massarsch, et al. (45)
64	Norwegian Clay	37	26	8	—	10.0	—	0.75	—	—	Bjerrum (10)
65	Saint-Alban Clay	65	45	22	60	27.0	8.9	0.70	0.47	1.000	Tavenas, et al. (73)
66	Moose River Muskeg	390	—	—	—	47.7	13.6	0.30	0.22	0.901	Adams (2)
67	Middleton Peat	510	—	—	—	57.4	8.0	0.31	0.18	0.998	Edil and Dhowian (19)
68	Portage Peat	600	—	—	—	53.8	16.0	0.30	0.09	0.998	Edil and Dhowian (19)
69	Fon du Lac Peat	240	—	—	—	50.2	8.0	0.53	0.18	0.998	Edil and Dhowian (19)
70	Kyoto Clay	—	88	57	52	32.5	—	0.45	—	—	Akai and Adachi (3)
71	Lagunillas Clay	—	61	37	30	26.8	—	0.53	—	—	Lambe (38,40)
72	Simple Clay	—	—	—	—	23.1	24.0	0.61	0.32	0.997	Ladd (34)
73	New England Marine Clay	—	—	20	—	32.0	16.0	0.50	0.41	0.995	Ladd (36)
74	Haney Clay	—	—	—	—	30.0	16.5	0.55	0.41	0.998	Campanella and Vaid (16)
75	Loess	21	35	11	18	31.5	6.3	0.36	0.54	0.983	Huergo (27)
76	Konnerud Clay	52	61	29	—	—	1.5	0.49	0.51	—	Bjerrum and Andersen (11)
77	Sundlund Clay	58	52	23	—	—	1.5	0.49	0.59	—	Bjerrum and Andersen (11)
78	Sterling Till	6	15	3	—	—	24.4	0.41	0.46	0.995	Murphy, et al. (49)
79	Gault Clay	—	85	55	68	—	4.0	0.75	0.27	0.989	Thompson (74)
80	Massachusetts Clay	—	—	23	—	32.7	—	0.48	0.45	—	Ladd (39)
81	Newfield Clay	—	31	13	32	28.6	20.0	0.50	0.28	0.996	Singh (66)

TABLE 2.—Summary of K_o Data for Sands during Virgin Load-Unload

Number (1)	Soil name (2)	Initial void ratio, e_o (3)	Relative density, D_r, as a percentage (4)	D_{60}, in millimeters (5)	Uniformity coefficient, C_u (6)	Effective friction angle, ϕ', in degrees (7)	Maximum OCR (8)	Earth pressure coefficient, K_{onc} (9)	Rebound exponent, α (10)	Sample correlation coefficient, r (11)	Reference (12)
82	Decomposed Granite	—	—	—	—	—	19.5	0.41	0.64	0.999	Pells (53)
83	Brasted Sand	—	40	—	—	—	74.1	0.43	0.27	0.983	Bishop (9)
84	Medium Sand	—	16	—	—	—	—	0.50	0.50	0.975	Bellotti, et al. (7)
85	Minnesota Sand	0.62	34	—	—	36.9	24.0	0.41	0.40	0.997	Hendron (24)
86	Reid-Bedford Sand	0.59	100	0.24	1.5	34.0	5.5	0.42	0.53	0.999	Al-Hussaini and Townsend (4,5)
87	Reid-Bedford Sand	0.68	72	0.24	1.5	32.6	—	0.45	—	—	Al-Hussaini and Townsend (4,5)
88	Reid-Bedford Sand	0.82	25	0.24	1.5	28.5	5.2	0.56	0.33	0.995	Al-Hussaini and Townsend (4,5)
89	Monterey No. 20 Sand	0.55	93	—	—	40.0	32.0	0.35	0.55	0.998	Wright (76)
90	Monterey No. 20 Sand	0.73	32	—	—	—	32.0	0.40	0.46	0.998	Wright (76)
91	Eastern Silica Sand	0.52	93	—	—	36.5	16.0	0.38	0.50	0.996	Wright (76)
92	Eastern Silica Sand	0.68	33	—	—	—	16.0	0.42	0.41	0.986	Wright (76
93	Ripley Sand	0.67	—	—	—	—	5.8	0.47	0.51	0.997	Menzies, et al. (46)
94	Glass Ballotini	0.56	100	0.1	—	36.5	62.5	0.38	0.26	0.997	Andrawes and El-Sohby (6)
95	Filter Sand	0.52	—	0.82	1.8	49.2	7.9	0.36	0.70	0.996	Weiler and Kulhawy (75)
96	Filter Sand	0.61	—	0.82	1.8	45.2	38.1	0.39	0.52	0.998	Weiler and Kulhawy (75)
97	Filter Sand	0.80	—	0.82	1.8	35.8	11.2	0.44	0.48	0.998	Weiler and Kulhawy (75)
98	Russian Sand	—	—	—	—	—	6.0	0.40	0.47	0.979	Fjodorov and Malychev (20)
99	Czechoslovakian Sand	—	—	—	—	—	11.3	0.41	0.49	0.995	Plelm (55)
100	German Sand	—	—	—	—	—	4.7	0.39	0.71	0.998	Mach (42)
101	German Standard Sand	0.67	—	1.0	1.0	35.0	42.9	0.53	0.44	0.983	Kjellman (31)
102	Kilyos Sand	0.64	47	0.15	1.25	28.0	—	0.52	0.39	—	Saglamer (59)
103	Ayvalik Sand	0.63	86	0.59	1.3	36.5	18.7	0.42	0.43	0.999	Saglamer (59)
104	Ayvalik Sand	0.75	47	0.59	1.3	33.5	18.9	0.47	0.45	—	Saglamer (59)
105	Ayvalik Sand	0.80	33	0.59	1.3	29.5	18.9	0.51	0.40	0.925	Saglamer (59)
106	Falgu Sandy Gravel I	0.72	88	1.9	1.5	36.5	3.7	0.39	0.72	0.999	Dayal, et al. (18)
107	Falgu Sandy Gravel II	0.91	4	3.6	1.4	33.0	4.6	0.37	0.69	0.997	Dayal, et al. (18)
108	Falgu Sandy Gravel III	0.68	87	6.0	1.5	41.0	6.3	0.25	0.78	0.999	Dayal, et al. (18)
109	Sangamon Sand	—	—	—	—	37.6	—	0.40	—	—	Al-Hussaini and Townsend (5)
110	Sangamon Sand	—	—	—	—	32.5	—	0.44	—	—	Al-Hussaini and Townsend (5)
111	Sangamon Sand	—	—	—	—	—	—	—	0.43	0.990	Holden (26)
112	Wabash Sand	—	—	—	—	38.6	—	0.39	—	—	Al-Hussaini and Townsend (5)
113	Wabash Sand	—	—	—	—	34.6	—	0.42	—	—	Al-Hussaini and Townsend (5)
114	Wabash Sand	—	—	—	—	—	—	—	0.41	0.980	Holden (26)
115	Chatahoochee Sand	—	—	—	—	40.5	—	0.44	—	—	Al-Hussaini and Townsend (5)
116	Chatahoochee Sand	—	—	—	—	37.2	—	0.44	—	—	Al-Hussaini and Townsend (5)
117	Chatahoochee Sand	—	—	—	—	33.5	—	0.49	—	—	Al-Hussaini and Townsend (5)
118	Chatahoochee Sand	—	—	—	—	32.3	—	0.49	—	—	Al-Hussaini and Townsend (5)
119	Brasted Sand	—	—	—	—	39.0	—	0.36	—	—	Al-Hussaini and Townsend (5)
120	Brasted Sand	—	—	—	—	33.9	—	0.46	—	—	Al-Hussaini and Townsend (5)
121	Sand	—	—	—	—	38.2	—	0.37	—	—	Al-Hussaini and Townsend (5)
122	Sand	—	—	—	—	37.0	—	0.42	—	—	Al-Hussaini and Townsend (5)
123	Sand	—	—	—	—	35.4	—	0.48	—	—	Al-Hussaini and Townsend (5)
124	Sand	—	—	—	—	32.9	—	0.54	—	—	Al-Hussaini and Townsend (5)
125	Belgium Sand	—	—	—	—	43.3	—	0.40	—	—	Al-Hussaini and Townsend (5)

TABLE 2.—Continued

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
126	Belgium Sand	—	—	—	—	40.2	—	0.40	—	—	Al-Hussaini and Townsend (5)
127	Belgium Sand	—	—	—	—	35.3	—	0.50	—	—	Al-Hussaini and Townsend (5)
128	Belgium Sand	—	—	—	—	34.2	—	0.50	—	—	Al-Hussaini and Townsend (5)
129	Minnesota Sand	—	—	—	—	37.5	—	0.33	—	—	Al-Hussaini and Townsend (5)
130	Minnesota Sand	—	—	—	—	28.0	—	0.38	—	—	Al-Hussaini and Townsend (5)
131	Pennsylvania Sand	—	—	—	—	35.8	—	0.40	—	—	Al-Hussaini and Townsend (5)
132	Pennsylvania Sand	—	—	—	—	31.0	—	0.51	—	—	Al-Hussaini and Townsend (5)
133	Pennsylvania Sand	—	—	—	—	—	—	—	0.42	0.980	Holden (26)
134	Ottawa Sand	0.57	73	0.42	2.1	42.7	30.0	0.49	0.69	0.990	Sherif, et al. (62)
135	Ottawa Sand	0.65	42	0.42	2.1	34.4	30.0	0.38	0.62	0.981	Sherif, et al. (62)
136	Ottawa Sand	0.75	4	0.42	2.1	25.0	30.0	0.58	0.53	0.979	Sherif, et al. (62)
137	Ottawa Sand	—	—	—	—	—	—	—	0.51	0.990	Holden (26)
138	Ottawa Sand 20-30	0.54	—	0.75	1.2	34.6	—	0.41	—	—	Edil and Dhowian (19)
139	Ottawa Sand 20-30	0.57	—	0.75	1.2	33.2	—	0.44	—	—	Edil and Dhowian (19)
140	Ottawa Sand 20-30	0.63	—	0.75	1.2	30.4	—	0.50	—	—	Edil and Dhowian (19)
141	Del Monte Sand	0.88	60	0.18	2.1	40.9	30.0	0.32	0.78	0.997	Sherif, et al. (62)
142	Del Monte Sand	0.98	41	0.18	2.1	34.3	30.0	0.36	0.76	0.998	Sherif, et al. (62)
143	Del Monte Sand	1.12	13	0.18	2.1	26.2	30.0	0.38	0.62	0.994	Sherif, et al. (62)
144	Mixture Sand	0.50	83	0.43	3.9	40.6	30.0	0.35	0.78	0.999	Sherif, et al. (62)
145	Mixture Sand	0.55	67	0.43	3.9	37.0	30.0	0.37	0.73	0.996	Sherif, et al. (62)
146	Mixture Sand	0.59	53	0.43	3.9	34.1	30.0	0.41	0.69	0.997	Sherif, et al. (62)
147	Mixture Sand	0.73	7	0.43	3.9	25.7	30.0	0.42	0.66	0.996	Sherif, et al. (62)
148	Highway 520 Sand	0.67	86	0.32	1.9	45.4	30.0	0.31	0.75	0.998	Sherif, et al. (62)
149	Highway 520 Sand	0.73	64	0.32	1.9	40.8	30.0	0.33	0.72	0.999	Sherif, et al. (62)
150	Highway 520 Sand	0.89	7	0.32	1.9	30.0	30.0	0.36	0.62	0.996	Sherif, et al. (62)
151	Golden Gardens Sand	0.68	77	0.50	1.8	43.5	30.0	0.32	0.84	0.997	Sherif, et al. (62)
152	Golden Gardens Sand	0.75	50	0.50	1.8	37.8	30.0	0.38	0.82	0.999	Sherif, et al. (62)
153	Golden Gardens Sand	0.81	27	0.50	1.8	33.8	30.0	0.38	0.73	0.997	Sherif, et al. (62)
154	Seward Park Sand	0.59	92	0.86	1.9	47.8	30.0	0.37	0.71	0.988	Sherif, et al. (62)
155	Seward Park Sand	0.63	75	0.86	1.9	44.3	30.0	0.40	0.67	0.995	Sherif, et al. (62)
156	Seward Park Sand	0.75	25	0.86	1.9	34.9	30.0	0.43	0.50	0.989	Sherif, et al. (62)
157	Sayers Pit Sand	0.62	71	0.69	2.3	38.8	30.0	0.36	0.64	0.990	Sherif, et al. (62)
158	Sayers Pit Sand	0.67	54	0.69	2.3	35.9	30.0	0.37	0.54	0.961	Sherif, et al. (62)
159	Sayers Pit Sand	0.77	18	0.69	2.3	30.7	30.0	0.37	0.54	0.986	Sherif, et al. (62)
160	Mathews Beach Sand	0.53	61	0.90	3.9	44.7	30.0	0.35	0.74	0.997	Sherif, et al. (62)
161	Mathews Beach Sand	0.59	42	0.90	3.9	38.2	30.0	0.37	0.73	0.999	Sherif, et al. (62)
162	Mathews Beach Sand	0.70	6	0.90	3.9	27.3	30.0	0.38	0.68	0.999	Sherif, et al. (62)
163	Alki Beach Sand	0.62	83	0.32	1.4	42.6	30.0	0.33	0.69	0.993	Sherif, et al. (62)
164	Alki Beach Sand	0.71	52	0.32	1.4	31.7	30.0	0.38	0.57	0.986	Sherif, et al. (62)
165	Alki Beach Sand	0.80	21	0.32	1.4	22.8	30.0	0.35	—	—	Sherif, et al. (62)
166	Pier 86 Sand	0.50	93	0.44	2.4	37.1	30.0	0.34	0.81	0.999	Sherif, et al. (62)
167	Pier 86 Sand	0.59	62	0.44	2.4	34.3	30.0	0.37	0.83	0.999	Sherif, et al. (62)
168	Pier 86 Sand	0.76	3	0.44	2.4	30.0	30.0	0.38	0.60	0.995	Sherif, et al. (62)
169	Ham River Sand	0.72	—	0.35	—	—	3.5	0.53	0.19	0.996	Daramola (17)
170	Ham River Sand	0.57	—	0.35	—	—	5.9	0.39	0.58	0.988	Daramola (17)
171	Edgar Sand	—	—	—	—	—	—	—	0.34	0.970	Holden (26)

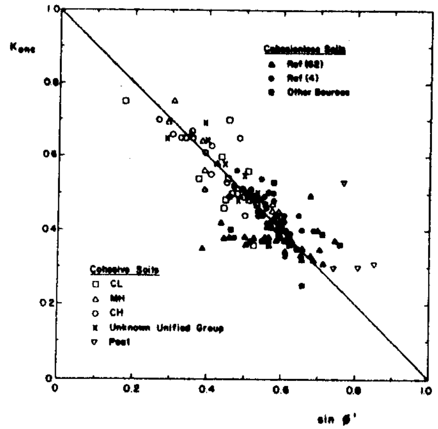

FIG. 2.—Observed Relationship between K_{onc} and sin ϕ' for Cohesive and Cohesionless Soils

extrapolated using dashed lines. Then, by definition

$$\alpha = \frac{\log(K_{ou}) - \log(K_{onc})}{\log(\text{OCR})} \quad \text{(7)}$$

for a range of values of OCR. The at-rest rebound parameter, α, is also the slope of the relationship between log (K_{ou}) and log (OCR). The mean values of α in Tables 1 and 2 have been determined from linear regression analyses for the soils considered, generally for values of OCR < 15. The sample correlation coefficients, r, are seen to be quite high, indicating that α appears to be constant with OCR.

Tavenas (72) has suggested that, as a reasonable upper limit, $\alpha \leq 1$. This seems intuitively correct since it cannot be expected to get more energy out of a soil than is put into it. Considering both clays and sands, α has a mean value of 0.509 with a standard deviation of 0.134.

Several investigators have suggested that the parameter α is related to the index properties of the soil. However, only vague trends were observed between α and plasticity index, clay fraction, liquid limit, or activity.

Schmidt (61) proposed that the parameter α is uniquely related to the effective stress friction angle, ϕ', of the soil. This approach appears to be substantiated by the general trend between α and sin ϕ', as shown in Fig. 5. The hypothesis taken is that

$$\alpha = \sin\phi' \quad \text{(8)}$$

which places theoretical upper and lower bounds on the at-rest rebound parameter such that $0 \leq \alpha \leq 1$. A statistical study of the data in Tables 1 and 2 revealed that

$$\alpha = 0.018 + 0.974 \sin\phi', \quad \text{(82 points)} \quad \text{(9a)}$$

$$\alpha = 0.929 - 0.852\, K_{onc}, \quad \text{(107 points)} \quad \text{(9b)}$$

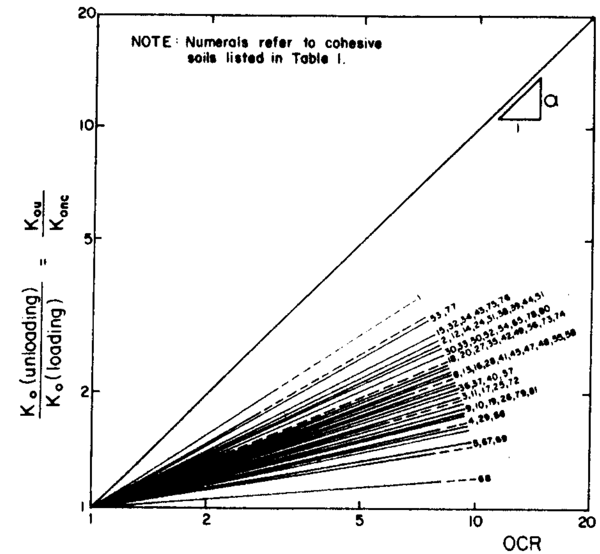

FIG. 3.—Trend between K_o and OCR for Cohesive Soils during Unloading

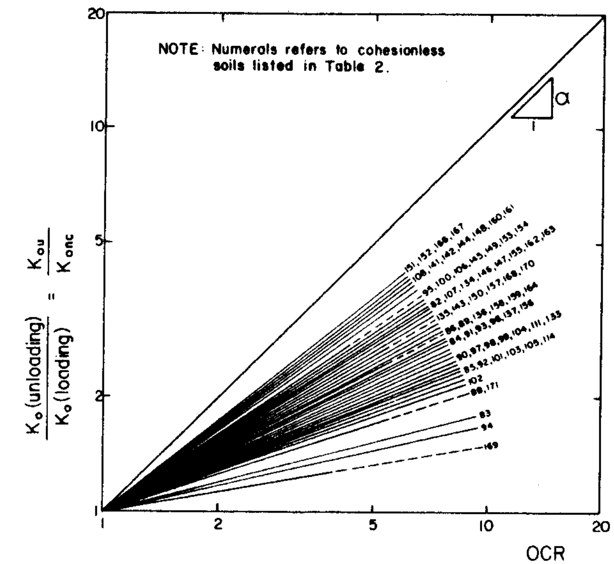

FIG. 4.—Trend between K_o and OCR for Cohesionless Soils during Unloading

which have sample correlation coefficients of 0.671 and 0.720, respectively. Since Eqs. 8 and 9a are approximately equal, the data suggest that K_o during loading-unloading simply may be related to ϕ' and OCR by

$$K_{ou} = (1 - \sin\phi')\,\text{OCR}^{\sin\phi'} \quad \text{(10)}$$

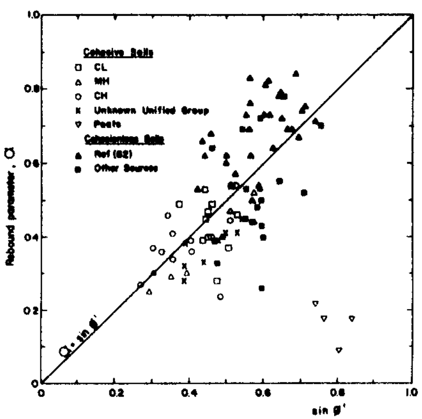

FIG. 5.—Relationship between At-Rest Rebound Parameter, α, and sin φ' for Clays and Sands

The application of Eq. 10 to four clays is shown in Fig. 6 and to four sands in Fig. 7.

Passive Failure.—The coefficient of passive earth pressure, K_p, may be assumed to be the upper limit on the value of K_{ou}. This defines a limiting value of OCR above which at-rest conditions do not apply and passive pressure is mobilized. For simplicity, a Rankine passive pressure coefficient can be adopted such that

$$K_p = \frac{1 + \sin \phi'}{1 - \sin \phi'} \qquad (11)$$

When $K_{ou} = K_p$ in Eq. 10, the limiting value of OCR for at-rest conditions is determined to be

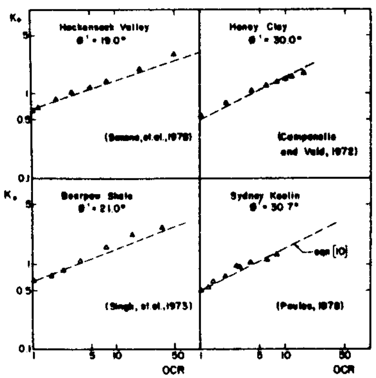

FIG. 6.—Measured and Predicted K_o of Four Clays during Loading-Unloading

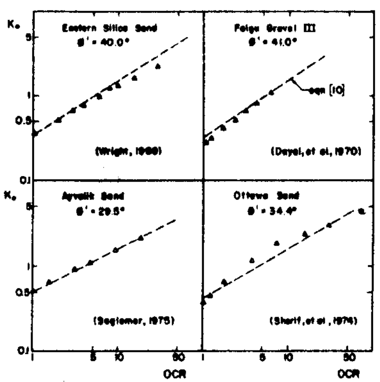

FIG. 7.—Measured and Predicted K_o of Four Sands during Loading-Unloading

$$\mathrm{OCR}_{\mathrm{limit}} = \left[\frac{(1 + \sin \phi')}{(1 - \sin \phi')^2}\right]^{(1/\sin\phi')} \qquad (12)$$

By reconstructing a geological history at Bradwell, Skempton (68) deduced a likely profile of K_o with depth for London Clay; values of K_o were reported to increase up to an OCR of about 25 and then decrease for higher values of OCR, suggesting passive failure. Using an effective friction angle, $\phi' = 20°$, for the Eocene clay, as determined by Skempton, Eq. 12 predicts that $\mathrm{OCR}_{\mathrm{limit}} = 27$.

Horizontal Stress During Reloading.—The little published data available on the behavior of K_o for soils during reloading are given in Table 3. Based on the trends observed with these 15 soils, an empirical approach may be formulated. Wroth (77) suggested that a linear relationship between σ'_h and σ'_v may be assumed, corresponding to the path CD in Fig. 1, such that

$$\sigma'_h - \sigma'_{h_{\min}} = m_r(\sigma'_v - \sigma'_{v_{\min}}) \qquad (13)$$

in which m_r = a constant termed the reload coefficient and $\sigma_{h_{\min}}$ and $\sigma_{v_{\min}}$ refer to point C in Fig. 1. If a new stress history parameter is defined as

$$\mathrm{OCR}_{\max} = \frac{\sigma'_{v_{\max}}}{\sigma'_{v_{\min}}} \qquad (14)$$

then, from Eq. 12, it can be shown that

$$\frac{\sigma'_{h_{\min}}}{\sigma'_{v_{\min}}} = K_{onc}(\mathrm{OCR}_{\max})^{\alpha} \qquad (15)$$

Then the value of K_o during reload, K_{or}, can be expressed as

TABLE 3.—Summary of K_o Data during Reload

Number (1)	Soil name (2)	m_r (3)	r (4)
11	Kaolin	0.43	0.999
26	Hackensack Clay	0.47	0.993
58	Khor Al-Zubair	0.36	0.990
74	Haney Clay	0.41	0.998
82	Decomposed Granite	0.34	0.988
83	Brasted Sand	0.36	0.998
86	Reid-Bedford Sand	0.27	0.989
88	Reid-Bedford Sand	0.43	0.996
90	Monterey Sand	0.26	0.995
95	Filter Sand	0.24	0.999
96	Filter Sand	0.23	0.999
97	Filter Sand	0.35	0.999
103	Ayvalik Sand	0.39	N/A
104	Ayvalik Sand	0.40	N/A
105	Ayvalik Sand	0.42	N/A

$$K_{or} = K_{onc}\left(\frac{OCR}{OCR_{max}^{(1-\alpha)}}\right) + m_r\left(1 - \frac{OCR}{OCR_{max}}\right) \quad \text{(16)}$$

The coefficient m_r was found to be a function of ϕ'; or alternatively as a function of K_{onc}, as shown in Fig. 8. The small data base suggests that

$$m_r = \left(\frac{3}{4}\right)(1 - \sin \phi') = \left(\frac{3}{4}\right)K_{onc} \quad \text{(17)}$$

By including the relationships given previously, one equation can be constructed to represent K_o as a function of stress history

$$K_o = (1 - \sin \phi')\left[\left(\frac{OCR}{OCR_{max}^{(1-\sin\phi')}}\right) + \frac{3}{4}\left(1 - \frac{OCR}{OCR_{max}}\right)\right] \quad \text{(18)}$$

Eq. 18 can be used to determine K_o anywhere along the stress paths shown in Fig. 1, and to determine the probable bounds of K_o in soil with more complex unload-reload histories. The approach requires that only the stress history (OCR and OCR_{max}) and ϕ' for a particular soil be known. For normally-consolidated soils, OCR_{max} = OCR = 1, and Eq. 18 reduces to Eq. 2. For overconsolidated soils during swelling or rebounding, OCR_{max} = OCR, and Eq. 18 is identical to Eq. 10. An application of Eq. 18 is shown in Fig. 9.

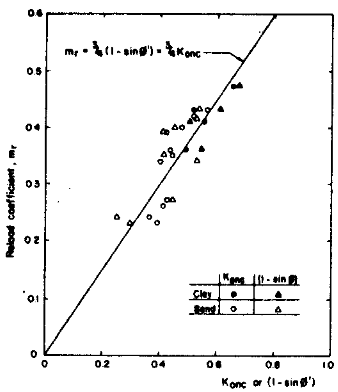

FIG. 8.—Trend between Reload Parameter m_r and K_{onc} or $\sin \phi'$

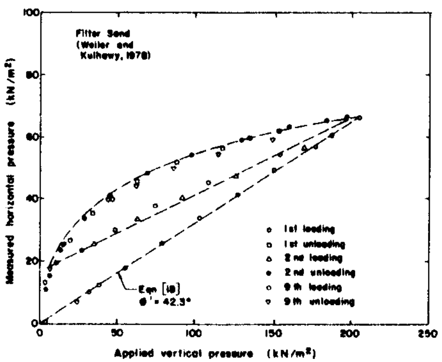

FIG. 9.—Measured and Predicted Response of Filter Sand (75) during Loading-Unloading-Reloading

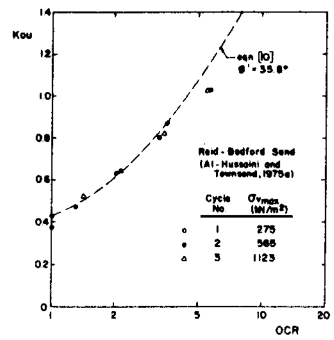

FIG. 10.—Observed K_o-OCR Relationship of Reid-Bedford Sand (5) for Three Load-Unload Cycles

For natural soils, the current value of OCR may be determined from conventional consolidation tests or other methods. At present, however, there appears to be no known technique of determining OCR_{max} for a specific soil deposit other than a good knowledge of local geology and stress history of the soil deposit.

Additional Considerations.—Some interpretation of available data by the writers was necessary to compile information as complete as possible. Generally, the soil data included in this study reflect soil parameters as reported by the respective authors. The effective stress friction angles cited are linear approximations to the failure envelopes over specific stress ranges. The actual failure envelopes are best represented by curved surfaces. In this study, no distinction has been made between ϕ' values determined from triaxial, direct shear, or simple shear devices.

One major problem in comparing the data is a consistent definition of effective stress friction angle. The most common alternative definitions used by the geotechnical community include: (1) Maximum deviator stress; and (2) maximum principal effective stress ratio. Which definition is most appropriate in the study of K_o still remains to be established. In addition, further research is needed to establish K_o behavior with regard to cyclic loading, rheological effects, residual soil deposits, gravels, and compacted fills.

Little is known about the effects of load-unload cycles on the value of K_o. The consequences of applying large numbers of cyclic loads on K_o remains to be investigated. For only a few cycles of load-unload, Eq. 10 still appears to be valid, as shown by Fig. 10. Within the applied stress ranges, different values of $\sigma_{v_{max}}$ had no appreciable effect on the K_{ou}-OCR relationship.

Conclusions

By reviewing laboratory data from over 170 different soils, it is established that K_o behavior during virgin compression, rebound, and reload can be represented approximately by simple empirical relationships. Statistical analyses are used to support the validity of the methods considered. The conclusions of this study are as follow:

1. The approximate theoretical relationship for K_{onc} of normally consolidated soils introduced by Jáky (28) appears valid for cohesive soils and moderately valid for cohesionless soils.
2. The variation of K_{ou} with OCR during unloading is approximately dependent on the effective stress friction angle of the material, ϕ', as suggested by Schmidt (61).
3. Horizontal stresses during reload may be estimated from a knowledge of ϕ' and the stress history (OCR and OCR_{max}).
4. The preceding relationships for K_o may be represented entirely by Eq. 18.

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Appendix.—References

1. Abdelhamid, M. S., and Krizek, R. J., "At Rest Lateral Earth Pressures of a Consolidating Clay," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 102, No. GT7, July, 1976, pp. 721–738.
2. Adams, J. I., "The Engineering Behavior of a Canadian Muskeg," *Proceedings*, 6th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Montreal, Canada, 1965, pp. 3–7.
3. Akai, K., and Adachi, T., "Study on the One-Dimensional Consolidation and Shear Strength Characteristics of Fully Saturated Clay," *Proceedings*, 6th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Montreal, Canada, 1965, pp. 146–150.
4. Al-Hussaini, M. M., and Townsend, F. C., "Stress Deformation of Sand under K_o Conditions," *Proceedings*, 5th Panamerican Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Buenos Aires, Argentina, 1975, pp. 129–136.
5. Al-Hussaini, M. M., and Townsend, F. C., "Investigation of K_o Testing in Cohesionless Soils," *Technical Report S-75-16*, Waterways Experiment Station, Vicksburg, Miss., Dec., 1975.
6. Andrawes, K. Z., and El-Sohby, M. A., "Factors Affecting K_o," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 99, No. SM7, July, 1973, pp. 527–539.
7. Bellotti, R., Formigoni, G., and Jamiolkowski, M., "Remarks on the Effects of Overconsolidation on K_o," *Proceedings*, Istanbul Conference on Soil Mechanics and Foundation Engineering, Vol. 1, 1976, Istanbul, Turkey, pp. 17–25.
8. Berre, T., and Bjerrum, L., "Shear Strength of Normally Consolidated Clays," *Proceedings*, 8th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1.1, Moscow, U.S.S.R., 1973, pp. 39–49.
9. Bishop, A. W., "Test Requirements for Measuring K_o," *Proceedings*, Brussels Conference on Earth Pressure Problems, Vol. I, 1958, Brussels, Belgium, pp. 2–14.
10. Bjerrum, L., "The Effective Shear Strength Parameters of Sensitive Clays," *Proceedings*, 5th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Paris, France, 1961, pp. 23–28.
11. Bjerrum, L., and Andersen, K., "In-situ Measurements of Lateral Pressures in Clay," *Proceedings*, 5th European Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Madrid, Spain, 1972, pp. 11–20.
12. Brooker, E. W., and Ireland, H. O., "Earth Pressures at Rest Related to Stress History," *Canadian Geotechnical Journal*, National Research Council, Ottawa, Ontario, Vol. 2, No. 1, Feb., 1965, pp. 1–15.
13. Brown, S. F., Andersen, K. H., and McElvaney, J., "The Effect of Drainage on Cyclic Loading of Clay," *Proceedings*, 9th International Conference on Soil Mechanics and Foundation Engineering, Vol. 2, Tokyo, Japan, 1977, pp. 195–200.
14. Burland, J. B., "Deformation of Soft Clay," thesis presented to Cambridge University, Emmanuel College, at Cambridge, England, in 1967, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
15. Calhoun, D. E., and Triandafilidis, G. E., "Dynamic Oedometer Study of Lateral Yield Effects," *Proceedings*, 7th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Mexico, 1969, pp. 65–72.
16. Campanella, R., and Vaid, Y., "A Simple K_o Triaxial Cell," *Canadian Geotechnical Journal*, National Research Council, Ottawa, Ontario, Vol. 9, No. 3, Aug., 1972, pp. 249–260.
17. Daramola, O., "On Estimating K_o for Overconsolidated Granular Soils," *Geotechnique*, Institute of Civil Engineers, London, England, Vol. 30, No. 3, Sept., 1980, pp. 310–313.
18. Dayal, U., Gairola, S. S. and Raju, V. S., "Coefficient of Earth Pressure at Rest of Granular Soils," *Journal of the Indian National Society of Soil Mechanics and Foundation Engineering*, Vol. 9, No. 4, 1970, pp. 371–386.
19. Edil, T. B., and Dhowian, A. W., "At-Rest Lateral Pressure of Peat Soils," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 107, No. GT2, Feb., 1981, pp. 201–220.
20. Fjodorov, I. V., and Malyshev, M. V., "O bokovomdavleniji v pescanych grunlach," *Gidrotechniceshoje Strojitelstvo*, Czechloslovakia, Vol. 23, No. 6, 1959, pp. 18–22.
21. Geotechnical Engineers, Inc., "Laboratory Testing Program for Bluff Stability Study," Boyd County, Nebraska, Charles T. Main, Inc., Nov., 1976.
22. Hanzawa, H., "Field and Laboratory Behavior of Khor-Al Zubair Clay, Iraq," *Soils and Foundations*, Japanese Society of Soil Mechanics and Foundation Engineering, Tokyo, Japan, Vol. 17, No. 4, Dec., 1977, pp. 17–30.
23. Hanzawa, H., "Geotechnical Properties of Normally Consolidated Fao Clay," *Soils and Foundations*, Japanese Society of Soil Mechanics and Foundation Engineering, Tokyo, Japan, Vol. 17, Dec., 1977, pp. 1–15.
24. Hendron, A. J., Jr., "The Behavior of Sand in One-Dimensional Compression," thesis presented to the University of Illinois, at Urbana, Champaign, 1963, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
25. Henkel, D. J., and Sowa, V., "The Influence of Stress History on Stress Paths in Undrained Triaxial Tests on Clay," *Laboratory Shear Testing of Soils, ASTM STP 361*, Ottawa, Canada, 1963, pp. 280–294.
26. Holden, J. C., "Research on Performance of Soil Penetrometers," Churchill Fellowship, 1971, C.R.B. of Victoria, Australia.
27. Huergo, P., "The Coefficient K_o in Remoulded Loess and Loam," *Proceedings*, 5th Panamerican Conference on Soil and Foundation Engineering, Vol. 1, Buenos Aires, Argentina 1975, pp. 281–290.
28. Jáky, J., "The Coefficient of Earth Pressure at Rest," *Journal for Society of Hungarian Architects and Engineers*, Budapest, Hungary, Oct., 1944, pp. 355–358.
29. Kelly, G. P., "An Evaluation of K_o in the Triaxial Test," *Internal Report*, Civil Engineering Department, Purdue University, West Lafayette, Ind., 1969.
30. Kinner, E., and Ladd, C. C., "Undrained Bearing Capacity of Footing on Clay," *Proceedings*, 8th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Moscow, U.S.S.R., 1973, pp. 209-215.
31. Kjellman, W., "Report on an Apparatus for Consumate Investigation of the Mechanical Properties of Soils," *Proceedings*, 1st International Conference on Soil Mechanics and Foundation Engineering, Vol. 2, Cambridge, Mass., 1936, pp. 16–20.
32. Knight, K., and Blight, G. E., "Studies of Some Effects Resulting from the Un-

loading of Soils," *Proceedings*, Sixth International Conference on Soil Mechanics and Foundation Engineering, Vol. 2, Montreal, Canada, 1965, pp. 103–107.
33. Kulkarni, R., "Effect of Structure on Properties of Marine Clay," *Proceedings*, 8th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1.1, Moscow, U.S.S.R., 1973, pp. 217–220.
34. Ladd, C. C., "Stress-Strain Behavior of Saturated Clay and Basic Strength Principles," *Rept. R64-17*, Massachusetts Institute of Technology, Cambridge, Mass., Apr., 1964.
35. Ladd, C. C., "Stress-strain Behavior of Anisotropically Consolidated Clays," *Proceedings*, 6th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Montreal, Canada, 1965, pp. 282–286.
36. Ladd, C. C., "Laboratory Determination of Soil Parameters for Excavation and Shallow Foundations," ASCE Proceedings Specialty Conference, *Field and Laboratory Determinations of Soil Parameters*, National Capital Section, Washington, D.C., 1976.
37. Ladd, C. C., Foott, R., Ishihara, K., Schlosser, F., and Poulos, H. G., "Stress-Deformation and Strength Characteristics," *Proceedings*, 9th International Conference on Soil Mechanics and Foundation Engineering, Vol. 2, Tokyo, Japan, 1977, pp. 421–494.
38. Ladd, C. C., and Lambe, T. W., "The Strength of an Undisturbed Clay Determined from Undrained Tests," *Laboratory Shear Testing of Soil, ASTM STP 361*, Ottawa, Canada, 1963, pp. 342–371.
39. Ladd, R. S., "Use of Electrical Pressure Transducers to Measure Soil Pressure," *Research Report RG5-48*, Dept. of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Mass., No. 180, pp. 1–79.
40. Lambe, T. W., "Methods of Estimating Settlement," *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 90, No. 9, Sept., 1964, pp. 47–70.
41. Leathers, F. D., and Ladd, C. C., "Behavior of an Embankment on New York Varved Clay," *Canadian Geotechnical Journal*, Vol. 15, No. 2, May, 1978, pp. 250–268.
42. Mach, V., "Laboratorri zkoumani vlivv opak. vaneho zatežovani sypkych zemin na hodnotu," Stavebricky časopis, Czechoslovakia, Vol. XVIII, No. 5, 1970, pp. 361–375.
43. Mahar, L. J., and Ingram, W. B., "Geotechnical Investigation for a Field Study of Pile Group Action," *FUGRO GULF Report 78-161-1 to FHWA*, October, 1979.
44. Massarsch, K. R., and Broms, B. B., "Lateral Earth Pressure at Rest in Soft Clay," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 102, No. GT10, Oct., 1976, pp. 1041-1047.
45. Massarsch, K. R., Holtz, R. D., Holm, B. G., and Fredriksson, A., "Measurement of Horizontal In-Situ Stresses," *Proceedings, Insitu Measurement of Soil Properties*, ASCE, Vol. 1, North Carolina State Univ., Raleigh, N.C., June, 1975, pp. 266-286.
46. Menzies, B., Sutton, H., and Davies, R., "A New System for Automatically Simulating K_o Consolidation and Swelling," *Geotechnique*, Vol. 27, No. 4, 1977, pp. 593–596.
47. Mitachi, T., and Kitago, S., "Change in Undrained Strength Characteristics of a Saturated Remolded Clay due to Swelling," *Soils and Foundations*, Vol. 16, No. 1, Mar., 1976, pp. 45–58.
48. Moore, P. J., and Cole, B. R., discussion of "At Rest Lateral Earth Pressure of a Consolidating Clay," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 103, No. GT7, July, 1977, pp. 820–821.
49. Murphy, D. J., Koutsoftas, D., Covey, J. N., and Fischer, J. A., "Dynamic Properties of Hard Glacial Till," *Earthquake Engineering and Soil Dynamics*, ASCE Proceedings, Vol. 2, Pasadena, Calif., June, 1978, pp. 636–659.
50. Parry, R. H. G. and Amerasinghe, S. F., "Components of Deformation in Clays," *Proceedings*, Symposium on Plasticity and Soil Plasticity, A. C. Palmer, Ed., Cambridge, England, 1973, pp. 108–126.
51. Parry, R. H. G., and Nadarajah, V., "Observations of Laboratory Prepared Lightly-Overconsolidated Specimens of Kaolin," *Geotechnique*, Vol. 24, No. 3, 1973, pp. 345–358.

52. Parry, R. H. G., and Wroth, C. P., "Pore Pressures in Soft Ground," *U.S. Army Engineer Waterways Experiment Station Rept. No. S-76-3*, Vicksburg, Miss., May, 1976.
53. Pells, P., "Stress Ratio Effects on Construction Pore Pressures," *Proceedings*, 8th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1.2, Moscow, U.S.S.R., 1973, pp. 327–332.
54. Pender, M. J., "A Model for the Behavior of Overconsolidated Soil," *Geotechnique*, Institute of Civil Engineers, London, England, Vol. 28, No. 1, 1978, pp. 1–25.
55. Plelm, H., "Rohrdruckversuche mit sandigen und kiesigen Erdstotten," Mitteilungen der Forschungsanstalt fur Schiffahrt, Wasser und Grundbau, Berlin, Germany, Heft 14, 1965, pp. 104–137.
56. Poulos, H. G., "Normalized Deformation Parameters for Kaolin," *ASTM Geotechnical Testing Journal, GTJODJ*, Vol. 1, No. 2, June, 1978, pp. 102–106.
57. Prévost, J., "Anisotropic Undrained Stress-Strain Behavior of Clays," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 104, No. GT8, Aug., 1978, pp. 1075–1090.
58. Průska, L., "Effect of Initial Stress on the Stress-Strain Relation," *Proceedings*, 8th International Conference on Soil Mechanics and Foundation Engineering, Vol. 4, Moscow, U.S.S.R., 1973, pp. 26–28.
59. Saglamer, A., "Soil Parameters Affecting Coefficient of Earth Pressure at Rest of Cohesionless Soils," *Proceedings*, Istanbul Conference on Soil Mechanics and Foundation Engineering, Istanbul, Turkey, 1975, pp. 9–16.
60. Saxena, S., Hedberg, J. and Ladd, C. C., "Geotechnical Properties of Hackensack Valley Varved Clays of N. J.," *ASTM Geotechnical Testing Journal, GTJODJ*, Vol. 1, No. 3, Sept., 1978, pp. 148–161.
61. Schmidt, B., discussion of "Earth Pressures at Rest Related to Stress History," *Canadian Geotechnical Journal*, National Research Council, Ottawa, Ontario, Canada, Vol. 3, No. 4, 1966, pp. 239–242.
62. Sherif, M. A., Ishibashi, I., and Ryden, D. E., "Coefficient of Lateral Earth Pressure At Rest in Cohesionless Soils," *Soil Engineering Research Report No. 10*, University of Washington, Seattle, Wash., Dec., 1974.
63. Sherif, M. A., and Koch, D. E., "K_o as Related to Soil Precompression Ratio," *Highway Research Record*, National Academy of Sciences, Washington, D.C., No. 323, 1970, pp. 39–48.
64. Sherif, M. A., and Strazer, R. J., "Soil Parameters for Design of Mt. Baker Ridge tunnel in Seattle," *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 99, No. SM1, Jan., 1973, pp. 111–122.
65. Simon, R. M., Christian, J. T., and Ladd, C. C., "Analysis of Undrained Behavior of Loads on Clays," ASCE Proceedings, *Analysis and Design in Geotechnical Engineering*, Vol. I, University of Texas, Austin, Tex., June, 1974, pp. 51–84.
66. Singh, H., "The Behavior of Normally Consolidated and Heavily Overconsolidated Clays at Low Effective Stresses," thesis presented to Cornell University, at Ithaca, N.Y., in 1971, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
67. Singh, H., Henkel, D. J., and Sangrey, D. A., "Shear and K_o Swelling of Overconsolidated Clay," *Proceedings*, 8th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1.2, Moscow, U.S.S.R., 1973, pp. 367–376.
68. Skempton, A. W., "Horizontal Stresses in an Overconsolidated Eocene Clay," *Proceedings*, 5th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Paris, France, 1961, pp. 352–357.
69. Skempton, A. W., and Sowa, V. A., "The Behavior of Saturated Clays During Sampling and Testing," *Geotechnique*, Institute of Civil Engineers, London, England, Vol. 13, No. 4, 1963, pp. 269–290.
70. Sketchley, C. J., and Bransby, P. L., "The Behavior of an Overconsolidated Clay in Plane Strain," *Proceedings*, 8th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Moscow, U.S.S.R., 1973, pp. 377–384.
71. Som, N. N., "The Effect of Stress Path on the Deformation and Consolidation of London Clay," thesis presented to the University of London, at London, England,

in 1968, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

72. Tavenas, F. A., "In-Situ Measurement of Initial Stresses and Deformation Characteristics," *Proceedings, Insitu Measurement of Soil Properties*, ASCE, Vol. 2, North Carolina State University, Raleigh, N.C., June, 1975, pp. 263–270.
73. Tavenas, F. A., Blanchette, G., Leroueil, S., Roy, M., and LaRochelle, P., "Difficulties in the In-Situ Determination of K_o in Soft Sensitive Clays," *Proceedings, Insitu Measurement of Soil Properties*, ASCE, Vol. 1, North Carolina State University, Raleigh, N.C., June, 1975, pp. 450–476.
74. Thompson, W. J., "Lateral Pressures in One-Dimensional Consolidation," *Proceedings*, 2nd Asian Regional Conference on Soil Mechanics and Foundation Engineering, Vol. 1, Japan, 1963, pp. 26–32.
75. Weiler, W. A., Jr., and Kulhawy, F. H., "Behavior of Stress Cells in Soil," *Contract Report B-49(4) to Niagara Mohawk Power Corporation, Geotechnical Engineering Report 78-2*, Cornell University, Ithaca, N.Y., Oct., 1978.
76. Wright, S. G., "A Study of Slope Stability and the Undrained Shear Strength of Clay Shales," thesis presented to the University of California, at Berkeley, Calif., in 1969, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
77. Wroth, C. P., "In-Situ Measurement of Initial Stresses and Deformation Characteristics," *Proceedings, Insitu Measurement of Soil Properties*, ASCE, Vol. 2, North Carolina State University, Raleigh, N.C., June, 1975, pp. 181–230.

Appendix II.—Notation

The following symbols are used in this paper:

d_{50} = 50% particle size;
ΔV = volume change;
ΔV_m = volume change due to membrane penetration; and
σ_3' = effective ambient stress.

Closure by Chin-Su Ting[3]

The writer wishes to thank Mr. R. W. Sarsby for his discussion and the valuable test data on the elimination of the influence of membrane penetration in triaxial compression tests. The membrane penetration effects on the test results of the volume change and the pore water pressure in triaxial tests on granular materials cannot indeed, be neglected and they should be corrected properly. Many investigators, such as S. Frydman and P. L. Newland (7,8), have studied meticulously the correction for volume change due to membrane penetration in triaxial tests on granular materials under hydraulic pressure. It seems that the following method proposed by the writer can be used for the correction for the error of volumetric strain due to membrane effects (7,8). The correction coefficient K_0 of the measured total volumetric strain during loading process can be expressed by $K_o = 1 - (A_m \cdot \Delta V^l_m/\Delta V_o)$ and the true total volumetric strain by $\epsilon'_{vo} = K_o \cdot \epsilon_{vo}$, ϵ_{vo} being the measured total volumetric strain before correction. Furthermore, if K_e and K_p be taken to express the correction coefficients of the elastic and plastic volumetric strains and $\epsilon^{e'}_{vo}$ and $\epsilon^{p'}_{vo}$ the true elastic and plastic volumetric strains respectively, their amounts can be derived from the following formulas: $K_e = 1 - (A_m \cdot \Delta v^u_m/\Delta v^e_o)$, $K_p = 1 - [A_m \cdot (\Delta v^l_m - \Delta v^u_m)/\Delta v^p_o]$ and $\epsilon^{e'}_{vo} = K_e \cdot \epsilon^e_{vo}$, $\epsilon^{p'}_{vo} = K_p \cdot \epsilon^p_{vo}$, ϵ^e_{vo} and ϵ^p_{vo} being the measured elastic and plastic volumetric strains before correction. In the preceding formulas, the following notations are adopted: Δv^l_m and Δv^u_m denote the influence value of volume change per unit area of contact between the membrane and the specimen during loading and unloading processes, respectively; Δv_o denotes the measured value of the total volume change under loading condition and Δv^e_o and Δv^p_o denote respectively the measured values of the elastic and plastic volume changes under unloading condition, finally, A_m denotes the contact area between the membrane and the specimen. It is assumed here that the contact area has the same value during loading and unloading processes. After making analysis on K_o, K_e and K_p, it is found that the values of all correction coefficients are usually less than one; in other words, the measured values of ϵ_{vo}, ϵ^e_{vo} and ϵ^p_{vo} are all greater than the true volumetric strains of ϵ'_{vo}, $\epsilon^{e'}_{vo}$ and $\epsilon^{p'}_{vo}$. The overestimation of the elastic volumetric strain is more serious in comparison with that of the plastic volumetric strain, because K_e is generally smaller than K_p. The overestimation percentage P^e and P^p of the elastic and plastic volumetric strains can be expressed as $P^e = (\epsilon^e_{vo} - \epsilon^{e'}_{vo})/\epsilon^{e'}_{vo} = [(1 - K_e)/K_e] \cdot 100(\%)$ and $P^p = (\epsilon^p_{vo} - \epsilon^{p'}_{vo})/\epsilon^{p'}_{vo} = [(1 - K_p)/K_p] \cdot 100(\%)$. It is an interesting fact that there will be no influence of membrane penetration on plastic volumetric strain when the values of Δv^l_m and Δv^u_m are substituted by the average influence value $\Delta \bar{v}_m$ in calculation. According to the above-mentioned analysis and using Fig. 8 and Eqs. 1 and 2 for specimens subjected to cyclic ambient pressure $\sigma_3 = 4.0$ kg/cm², the test results given in Mr. Sarsbys discussion, the writer has calculated the correction coefficients, the true volumetric strains and the overestimation as a percentage of the volumetric strains for tested sample with $r_d = 1.70$ g/cm³ under hydraulic pressure $\sigma_3 = 4.0$ kg/cm². The tested sand specimen has $d_{50} = 0.28$ mm, $A_m = 98.81$ cm², $\Delta v_o = 1.89$ cm³, $\Delta v^e_o = 1.09$ cm³, $\Delta v^p_o = 0.8$ cm³, $\epsilon_{vo} = 2.011\%$, $\epsilon^e_{vo} = 1.159\%$, $\epsilon^p_{vo} = 0.852\%$, and the amounts of Δv^l_m and Δv^u_m calculated from Eqs. 1 and 2 are respectively 48.50 cm³/m² and 30.16 cm³/m². From these data, the correction coefficients $K_o = 0.746$, $K_e = 0.727$, $K_p = 0.774$, the true volumetric strains $\epsilon'_{vo} = 1.50\%$, $\epsilon^{e'}_{vo} = 0.84\%$, $\epsilon^{p'}_{vo} = 0.66\%$. By calculation, we find the error caused by membrane penetration for the total volumetric strain is (2.011 − 1.50)% = 0.511%, that for the elastic volumetric strain is (1.159 − 0.84)% = 0.319% and that for the plastic volumetric strain is (0.852 − 0.66)% = 0.192%, and the overestimation in percentage of elastic and plastic volumetric strains are 38% and 29.2%, respectively.

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Appendix.—References

7. Frydman, S., Zeitlen, J. G., and Alpan, I., "The Membrane Effect in Triaxial Testing of Granular Soils," *Journal of Testing and Evaluation*, JTEVA, Vol. 1, No. 1, Jan., 1973.
8. Newland, P. L., and Allely, B. H., "Volume Changes in Drained Triaxial Tests on Granular Materials," *Geotechnique*, Vol. 9, 1959.

K_o-OCR Relationships in Soils[a]

Discussion by R. Bellotti,[3] V. Ghionna,[4] and M. Jamiolkowski[4]

The authors have presented a comprehensive review concerning existing experimental evidence relating K_o to the stress history (OCR) and contemplating both, first unloading from the virgin compression curve and first reloading towards the virgin compression curve. The writers, who have been involved since the early seventies in experimental research with the aim to assess through laboratory (7) and in-situ tests

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TABLE 4.—Summary of K_o Data for Ticino Sand as Obtained from ENEL-CRIS Calibration Chamber Tests (78)

D_R, as a percentage (1)	D_{50}, in millimeters (2)	C_u (3)	OCR (4)	K_{onc} (5)	α (6)	r (7)	Number (8)	Note (9)
91	0.59	1.58	2–7	0.411	0.523	0.999	7	Dry
72	0.59	1.58	2–8	0.427	0.480	0.999	19	Dry
37	0.59	1.58	2–9	0.456	0.410	0.999	17	Dry
90	0.59	1.62	5	0.406	0.434	0.999	1	Saturated

(80) the horizontal stress existing in natural soil deposits, would like to present some additional experimental data concerning sands and make a few comments which may be of some interest to the authors and to the readers of this paper.

The writers are conducting a calibration of the electrical CPT and self-boring pressuremeter in pluvially deposited Ticino sand (78) using a calibration chamber housing specimens 1.2 m in diam and 1.5 m in height. During this experimental work a large number of specimens having the desired stress history have been created through one-dimensional compression and eventual one-dimensional rebound under strictly controlled boundary conditions (78). During these tests by careful measurements of σ'_h and σ'_v it was possibly to assess the values of K_{onc}, K_{ou} and α. The summary of the obtained experimental results is shown in Table 4 which gives also some information concerning the Ticino sand.

For this sand the strength envelope is curvilinear and may be described by the failure criterion proposed by Baligh (79). The parameters ϕ_o and α_o defining this failure criterion are given in the Table 5. ($\phi_o = \phi$ for 2.72 reference stress equal to 10 t/m^2.)

These experimental data seem to support very well the empirical formulae reported by the authors. The writers believe that the moderate validity of the Jaky (28) relationship for K_{onc} mentioned in the author's conclusions should in some way be related to the difficulty in determining the relevant value of ϕ' due to the nonlinear strength envelope of many tested sands considered in Table 2 of the author's paper.

Similar results to those reported here above for Ticino sand have been obtained for the Hokksund sand at the Norwegian Geotechnical Institute (81) following the same testing procedure and using a calibration chamber equal to the one used in the research on Ticino sand. The relevant experimental results for Hokksund sand are summarized on Table 6.

For Hokksund sand, the parameters which describe the curvilinear strength envelope in the dense state are ϕ_o = 41.4° and α_o = 6°.

TABLE 5.—Parameters Defining Strength Envelope of Ticino Sand

D_R, as a percentage (1)	ϕ_o, in degrees (2)	α_o, in degrees (3)
91	43.3	9.1
72	40.6	9.0
37	38.6	5.7

TABLE 6.—Summary of K_o Data for Hokksund Sand as Obtained at NGI (81) from Calibration Chamber Tests (79)

D_R, as a percentage (1)	D_{50}, in millimeters (2)	C_u (3)	OCR (4)	K_{onc} (5)	α (6)	r (7)	Number (8)	Note (9)
92	0.405	1.87	6.5	0.353	0.455	0.964	6	Dry
80	0.405	1.87	7	0.343	0.423	0.957	12	Dry
60	0.405	1.87	7	0.362	0.356	0.968	4	Dry
26	0.405	1.87	7	0.414	0.299	0.968	10	Dry
9	0.405	1.87	7	0.437	0.273	0.978	3	Dry

Appendix.—References

78. Baldi, G., Bellotti, R., Ghionna, V., Jamiolkowski, M., and Pasqualini, E., "Cone Resistance in Dry NC and OC Sands," Proc. of a Session on "Cone Penetration Testing and Experience," Geotech. Engrg. Div., ASCE National Convention, St. Louis, 1981.
79. Baligh, M. M., "Theory of Deep Site Static Cone Penetration Resistance," MIT Report N°. R-75-76, Boston, 1975.
80. Ghionna, V., Jamiolkowski, M., Lancellotta, R., Tordella, M. L., and Ladd, C. C., "Performance of Self-Boring Pressuremeter Tests in Cohesive Deposits," Report for U.S. Dept. of Transportation, FHWA, Contract N° Dot-FH-11-9264, 1981.
81. Parkin, A., Holden, J., Aamot, K., and Lunne, T., "Laboratory Investigations of CPT's in Sand," NGI Internal Report N°. 5210-8-9, 1980.

Discussion by Tuncer B. Edil,[5] M. ASCE

The authors have been thorough in collecting the existing K_o data. Statistical interpretation of such extensive data is useful in delineating general behavioral trends which may be otherwise obscured by testing procedures and variabilities. This is particularly true for a property such as K_o which is difficult to measure and sensitive to the measurement technique. However, such empirical equations could be misleading if limitations are not clearly indicated. For peat soils as a group, there is no apparent correlation between K_o and α (rebound parameter) and sin ϕ' as shown in Figs. 2 and 5. Therefore, Eq. 18 is not valid for such soils and indeed would result consistently in lower values of K_o and higher values of α based on their friction angle. This is a result of the effect of fibers encountered in peat and certain industrial sludges (such as paper mill sludge).

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Discussion by Richard L. Handy,[6] M. ASCE

The collection, presentation, and analysis of available K_o data is a substantial contribution that calls for weighty, if not grandiose, discussion.

The authors' conclusion that the Jáky relationship (28) for K_o appears valid should be clarified, since correlation analysis, while appropriate to define a trend, does not establish the reliability of that trend for prediction of either average or individual values needed for design. Regression is the more appropriate statistical method for prediction, wherein confidence bands may be defined to show ± confidence limits either for means or for individual values (85).

To illustrate, a rough sketch of a band to include half of the points of Fig. 2 suggests about a 50% likelihood of an individual value of K_o coming within about ±0.1 of the central trend line—not very good for a predictor when all values are roughly 0.5 ± 0.25. Furthermore, for cohesionless soils, $r^2 = 0.39$ indicates that 61% of the variation in K_o cannot be attributed to variations in sin ϕ.

Sandpile Factor.—Engineers commonly assume that the Jáky equation defines lateral stress ratio in normally consolidating soil under a broad, level, loaded area. As derived, however, the equation defines stress ratio at the center of a pile of sand whose surfaces are inclined at the angle of repose, ϕ (28). This misunderstanding suggests a clear disadvantage to publishing in Hungarian, although the pile geometry was critically reviewed some 30 yr ago by Tschebetarioff (87).

Jáky's figure caption translates (28), "Slipping shoulders on a motionless soil mass," with a clarity of expression that almost compensates for the previously cited disadvantage of publishing in Hungarian. His sketch shows sloping shoulders with internal shear planes oriented at ϕ and at 90° to horizontal. The central core of stable sand is bounded by σ_3 planes inclined 45° + $\phi/2$ to horizontal, so what ordinarily is thought of as a "K_o condition," with σ_3 horizontal, exists only at the center of the core. By ignoring bin effect and assuming that vertical stress equals overburden pressure, Jáky obtained at the center (28)

$$K_o = (1 - \sin\phi)\frac{1 + \frac{2}{3}\sin\phi}{1 + \sin\phi} \approx 0.9\,(1 - \sin\phi), \quad (19)$$

which he later rounded off (83) to: $K_o = 1 - \sin\phi$. Although Jáky refers to this in English as the "coefficient of earth pressure at rest," he used it for analysis of pressures in silos with vertical wall friction that simulates the vertical shear planes envisioned in the sandpile shoulders (83).

Thus, it would appear that experimental testing of Jáky's theory properly should be performed in a pile of sand or in a silo, not in a smooth-sided consolidometer. Why then, does it repeatedly test out?

Mechanism of K_{onc}.—K_{onc} must represent wedging apart of soil grains in a lateral direction by intrusion of other grains vertically during consolidation. The net decrease in soil volume is in direct contrast to volume increases during shear testing, where dilatancy often contributes positively to ϕ'. Thus it should not be surprising that the Jáky relationship of K_{onc} to ϕ' is less reliable for granular soils than for cohesive soils, because of the difference in their dilatancy.

In apparent recognition of this, Tschebetarioff argued some years ago that K_o should depend on sliding friction alone, without an interlocking component (87). He then rejected this hypothesis on the basis that his measured coefficients of sliding friction for different minerals were more variable than is K_o (87,88). Let us re-examine what should be a viable concept, that K_{onc} depends on resistance to sliding without dilatancy, which is always negative during consolidation but typically positive during shear tests.

By analogy to the Rankine expression we may write (83)

$$K_{onc} = \frac{1 - \sin\phi_s}{1 + \sin\phi_s} \quad (20)$$

in which ϕ_s = angle of sliding friction. The ϕ_s for quartz is about 25° (84), which gives $K_o = 0.40$, whereas in clays the nondilatent residual strength ϕ_r = 9° to 21° (84), which gives K_o in the range 0.45 to 0.25—all in rather good agreement with average values from Table 1.

Particle Shape and Packing.—Equation 20 requires that slip can occur unimpeded at an optimal angle, 45° + $\phi_s/2$ from horizontal. But as densification continues, particle interference may challenge that angle and divert it into less favorable orientations, adding to ϕ_s and decreasing K_o. Thus,

$$K_{onc} = \frac{1 - \sin(\phi_s + \alpha)}{1 - \sin(\phi_s + \alpha)} \quad (21)$$

in which α represents diversion of the slip angle. α will depend on depletion of favorable slip orientations, which in turn relates to packing geometry that incidentally affects dilatant behavior during shear. If this is correct, the Jáky relationship succeeds somewhat by accident and cannot be expected to be an accurate predictor.

K_o during rebound is even more complicated: The optimal slip direction shifts to 45 − $\phi_s/2$, reducing α, as required for rebound slip. Because of this and other unknown factors of OCR and clay expansion, it appears the surest way to ascertain K_o in the field may be to measure it (82).

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Appendix.—References

82. Handy, R. L., Remmes, B., Moldt, S., Lutenegger, A., and Trott, G., "In Situ Stress Determination by Iowa Stepped Blade," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 108, No. GT11, Nov., 1982, pp. 1405–1422.
83. Jáky, J., "Pressure in Silos," *Proceedings*, 2nd International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, 1948, pp. 103–107.

84. Mitchell, J. K., *Fundamentals of Soil Behavior*, John Wiley and Sons, Inc., New York, N.Y., 1976.
85. Snedecor, G. W., *Statistical Methods*, Iowa State University Press, 5th edition, Ames, Iowa, 1956, pp. 139.
86. Spangler, M. G., and Handy, R. L., *Soil Engineering*, Intext, New York, N.Y., 3d edition, 1973, pp. 474; 4th edition, Harper and Row, N.Y., 1982, pp. 469.
87. Tschebotarioff, G., *Soil Mechanics, Foundations, and Earth Structures*, McGraw-Hill Book Co., New York, N.Y., 1951, pp. 256.
88. Tschebotarioff, G., and J. D. Welch, "Lateral Earth Pressures and Friction between Soil Minerals," *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Engineering, Vol. 7, 1948, pp. 135–138.

Discussion by M. Jamiolkowski[7] and E. Pasqualini[7]

The writers wish to contribute to the stimulating and useful review of the empirical correlations linking K_o to ϕ and OCR, the validity of which, for a world-wide variety of soils, has been well-documented by the authors. The first aim of this discussion is to add some more up-to-date experimental data on the Italian cohesive soils mentioned in Table 1 which have been taken from Bellotti, et al. (7). The writers have continued their systematic evaluation of K_o, K_{ou}, and K_{or} using two types of instrumented oedometer rings as has been described previously.

The resulting experimental results are summarized in Table 7.

An examination of the details of the preceding experimental results allows the following comments:

1. On average, the ratio of $K_{onc}/1 - \sin\bar{\phi}$ is equal to 1.02 ± 0.17.
2. The ratio of the exponent α over $\sin\bar{\phi}$ is equal to 0.87 ± 0.18 for all examined soils, indicating that for the tested clays α tends to be lower than $\sin\bar{\phi}$.
3. The ratio of the reloading coefficient m_r to K_{onc} varies, for the tested soils, between 0.8 and 1.0 and is equal to 0.90 ± 0.08 on average, slightly higher than the values postulated by the authors.
4. From the writers' experimental data it appears that, for a series of subsequent unloading and reloading cycles with monotonically increasing OCR_{max}, the observed α and m_r values are subject only to minor changes and may be considered as constant for all practical purposes.

TABLE 7.—Summary of K_o Data during Virgin Loading-Unloading-Reloading

Site (1)	W_L, as a percentage (2)	PI, as a percentage (3)	$\bar{\phi}$, in degrees (4)	Maximum OCR_u (5)	α (6)	r_u (7)	N_u[a] (8)	Maximum OCR_m (9)	m_r (10)	r_m (11)	N_r[b] (12)	K_{onr} (13)
Porto Tolle	52 ± 2	30 ± 2	29	10–32	0.41 ± 0.06	0.992 ± 0.01	11	10–48	0.49 ± 0.06	0.999 ± 0.00	6	0.52 ± 0.02
Tarquinia	52 ± 6	34 ± 6	27	12–50	0.42 ± 0.05	0.995 ± 0.00	14	8–32	0.53 ± 0.05	0.999 ± 0.00	7	0.57 ± 0.02
Trieste	71 ± 12	47 ± 10	26	4–28	0.53 ± 0.03	0.995 ± 0.00	7	16–24	0.48	0.997	2	0.57 ± 0.03
Panigaglia	75 ± 5	55 ± 10	26	6–46	0.48 ± 0.08	0.994 ± 0.00	5	24–40	0.48	0.996	3	0.63 ± 0.02
Cagliari	42 ± 1	25 ± 1	—	24–32	0.52 ± 0.02	0.996 ± 0.00	2	48	0.52	0.999	1	0.58 ± 0.3

[a]N_u = number of available unloading curves.
[b]N_r = number of available reloading curves.

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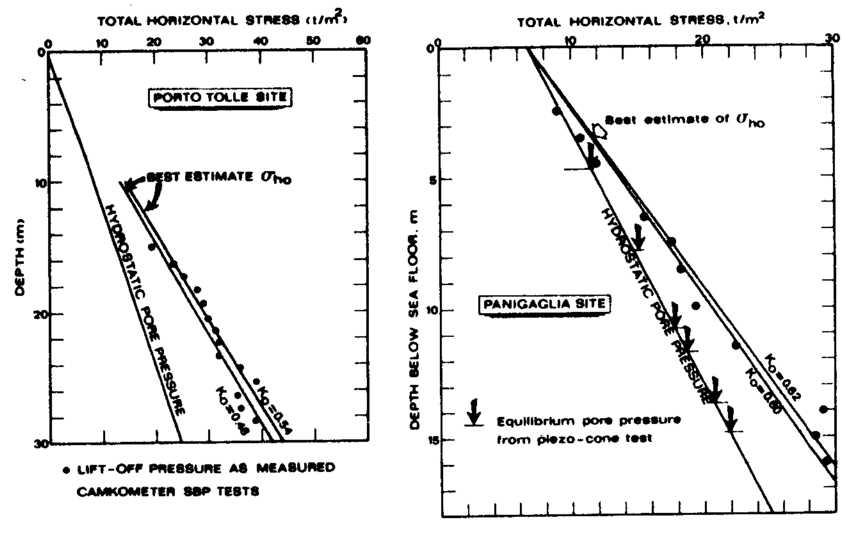

FIG. 11.—Total Horizontal Stress Measured during Self-Boring Pressuremeter Tests

The validity of the laboratory-determined K_{onc} values may be, at least partly, inferred from a comparison between estimated in situ total horizontal stress, σ_{ho}, and the values measured during the self-boring pressuremeter tests (90,91,92) carried out at the Porto Tolle and Panigaglia sites, see Fig. 11.

The estimated σ_{ho} has been computed using laboratory measured K_{onc} values and referring to the known hydrostatic pore pressure.

From a general point of view, in addition to the considerations made by the authors on the definition of the $\bar{\phi}$ values to be used in Eqs. like 2–5, 9*a*, and 10 it may be useful to add for the sake of clarity that: (1) $\bar{\phi}$ should be intended as peak angle of shearing resistance in terms of effective stress as obtained from consolidated-drained (CD) triaxial compression tests on high-quality undisturbed specimens of NC soils; and (2) when such tests are not available, the value of $\bar{\phi}$ may be inferred from consolidated-undrained (CU) triaxial compression tests with pore pressure measurement; in this case, the failure criterion will preferably be referred to the maximum difference of principal stresses (89).

Even within this frame the value of $\bar{\phi}$ is not defined unequivocally, thus in many natural NC aged or structural cohesive deposits $\bar{\phi}$ (CU), or both, may result to be larger than $\bar{\phi}$ (CD) (93).

ACKNOWLEDGMENT

The writers wish to express their gratitude to Dr. E. Ruberl and Mr. U. Pavesi from S.G.I., Milan, who performed laboratory tests presented in this paper.

Appendix.—References

89. Bjerrum, L., and Simons, N. E., "Comparison of Shear Strength Characteristics of NC Clays," *Proceedings*, Research Conference on Shear Strength of Cohesive Soils, ASCE, Boulder, Colo., 1960.
90. Ghionna, V., Jamiolkowski, M., and Lancellotta, R., "Characteristics of Saturated Clays as Obtained from SBP Tests," *Proceedings*, Symposium on the Pressuremeter and its Marine Application, Paris, France, 1982.
91. Ghionna, V., Jamiolkowski, M., and Lancellotta, R., Discussion, *Proceedings*, Symposium on the Pressuremeter and its Marine Application, Paris, France, 1982.
92. Ghionna, V., Jamiolkowski, M., Lancellotta, R., Tordella, M. L., and Ladd, C. C., "Performance of Self-Boring Pressuremeter Tests in Cohesive Deposits," Report for U.S. Dept. of Transportation, FHWA, *Contract No. Dot-FH-11-9264*, 1981.
93. Leonards, G. A., "Stability of Slopes in Soft Clays," General Report, *Proceedings*, Sixth Panamerican Conference on SM and FE, Lima, Peru, 1979.

Discussion by Birger Schmidt,[8] M. ASCE

This paper provides a valuable confirmation of Jáky's equation for the at-rest pressure coefficient during primary loading, K_{onc}, and Schmidt's equation for the at-rest coefficient for rebound or unloading, K_{ou}. It is gratifying that such relatively complex relationships can be expressed in very simple terms, using only one soil parameter, applicable to all soil types. The writer offers the following comments to the paper.

Figure 5 shows that the peats conform very poorly to the average relationship of $\alpha = \sin\phi'$. Figure 2 shows a more modest lack of conformance by peat to the relationship $K_{onc} = 1 - \sin\phi'$. It is suggested that the effect of fibers and directional properties of many peats is responsible for this lack of conformance, and that the equation should be used only for homogeneous and nonfibrous peat, if for any peat.

Table 1 shows a number of tests with OCR values substantially greater than 30. In theory, these samples should have failed in passive failure, yet sample correlation coefficients are satisfactory, suggesting a linear relationship also through the high OCR range. Did any of these tests show a fall-off of K_{ou} for high OCR? If not, one might speculate that the reason is a contribution to the passive strength from an effective cohesion. Alternatively, test peculiarities could be responsible. It should also be considered that all of these tests are short-term laboratory tests; aging and relaxation with time could reduce these horizontal stresses to a point below the cohesionless Rankine value.

The generalized equation (Eq. 18) applicable to the reload curve is interesting but it has a flaw, in the writer's opinion. Experience and common sense would suggest that the horizontal pressure during reload would approach the virgin curve at a point close to the previous maximum pressure; i.e., at OCR = 1. Equation 18 suggests a family of parallel curves (straight lines) for different values of OCR_{max}, none of which join the virgin curve at OCR = 1, though they generally come close. The writer proposes the following simpler equation in lieu of Eq. 18:

$$K_o = \frac{1 - \sin\phi'}{OCR_{max} - 1}\left(OCR_{max} - OCR + (OCR - 1)\, OCR_{max}^{\alpha}\right) \qquad (22)$$

which describes a straight line between the points of $OCR = OCR_{max}$ and OCR = 1; i.e., between the minimum and maximum stresses.

Many tests have shown that K_o in reload is not necessarily linear with OCR, so neither Eq. 18 nor Eq. 22 can be taken as more than an approximation.

Incidentally, Eq. 6 was developed almost simultaneously and independently by Schmidt (61) and Alpan (94), who also presented an interesting examination of the limiting state of passive failure.

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Appendix.—Reference

94. Alpan, I., "The Empirical Evaluation of the Coefficient K_o and K_{or}," *Soil and Foundations*, Vol. 7, No. 1, Tokyo, Japan, 1967, pp. 31–40.

Closure by Paul W. Mayne[9] and Fred H. Kulhawy[10]

The writers appreciate the general interest and responses provided by the discussers. The additional data on clays presented by Jamiolkowski and Pasqualini and data on sands by Bellotti, Ghionna, and Jamiolkowski are truly welcome. These laboratory and field studies are greatly needed towards understanding the true and complex behavior of K_o in situ. The nonlinearity of the strength envelope, as well as the assumed linearity in each of the simple relationships used to describe the K_o stress paths (load, unload, or reload), could indeed account for scatter in the observed trends.

Both Schmidt and Edil draw attention to the fact that peats behave differently than other soil types. Data presented by Adams (2) and Edil and Dhowian (19) support this argument.

Schmidt hypothesizes that aging (secondary compression) may possibly reduce the magnitude of horizontal stresses. Other researchers (9,14,77,95,97) however, claim that K_o should remain constant with time. In addition, Schmertmann (98) has recently posed this question rhetorically.

With regard to Eq. 19 proposed by Schmidt, Wroth (77) describes that the K_o stress path during reload achieves a value K_{onc} before the maximum preconsolidation stress. The observed and predicted (Eq. 18) behavior of two clays shown in Fig. 12 substantiate the concept of a hys-

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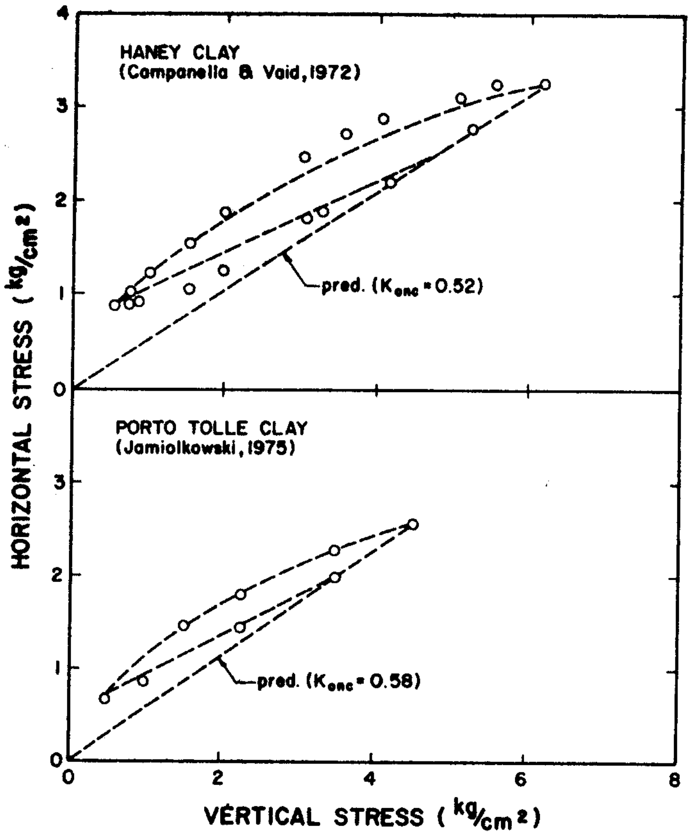

FIG. 12.—Comparison of Measured and Predicted K_o Behavior for Haney Clay (16) and Porto Tolle Clay (96) during Load-Unload-Reload Conditions

teresis, not necessarily closing at the preconsolidation stress. Moreover, reviewing the meager data base available on K_o during reloading, researchers have shown the K_o-reload path intersects the K_{onc} line below (5,9,16,75,76,96), at (66,75), or above (5,53) the original value of maximum preconsolidation pressure. (Data from Refs. 22, 59, and 60 were not complete enough to determine this behavior.) Perhaps, Jamiolkowski and his fellow writers to this discussion could provide additional data in clarifying this issue.

Handy questions the validity of Jáky's theoretical derivation, as also criticized by others (1,6,18,24,47), and presents an expression for K_{onc} in terms of "true" or "residual" friction angle. More theoretically sound methods for predicting K_o have also been proposed by others (14,24,47, 54,58,77) usually at the expense of simplicity.

As an alternative to estimating K_o, Handy recommends the actual field measurement of K_o. Although the in situ determination of lateral stress is becoming more attractive with a variety of recent field techniques, these methods have their own inherent problems and are not economically justifiable on all geotechnical projects. Equation 18 was developed by the writers to allow a first-order estimate of K_o in situ.

Appendix.—References

95. Hanzawa, H., and Kishida, T., "Fundamental Considerations on Undrained Strength," *Soils and Foundations*, Vol. 21, No. 1, Mar., 1981, pp. 39–50.
96. Jamiolkowski, M., discussion in *Proceedings*, Istanbul Conference on Soil Mechanics and Foundation Engineering, Vol. 3, Istanbul, Turkey, 1975, pp. 54–55.
97. Mayne, P. W., discussion to "Triaxial Testing at NGI," by T. Berre," *Geotechnical Testing Journal*, ASTM, Dec., 1982.
98. Schmertmann, J. H., "A Simple Question about Consolidation," *Journal of Geotechnical Engineering*, ASCE, Vol. 109, No. 1, Jan., 1983, pp. 119–122.

Permeability and Consolidation of Normally Consolidated Soils[a]

Discussion by Eulalio Juarez-Badillo,[4] F. ASCE

The work realized by the authors is in an interesting area at present. The writer has great interest in better equations than those that have been traditional in soil mechanics and has made some work on it. On the interest of finding and trying to establish the better equations, the writer wants to comment and compare with his own relationships the equations presented by the authors.

For the k and e relationship the authors present Eq. 3

$$k = C \frac{e^n}{1 + e} \qquad (3)$$

The corresponding equation found by the writer (17) is

$$k = k_o (1 + e)^\kappa \qquad (15)$$

In Eq. 3, $k = C/2$ for $e = 1$, and $k = 0$ for $e = 0$. In Eq. 15, $k = k_o$ for $e = 0$, that is, for the soil when it has been compressed to a volume equal to the "initial volume of solids." Equation 15 very nicely satisfies both experimental data and a philosophic requirement. κ has been called the "coefficient of permachange."

For the c_v and $\bar{\sigma}$ relationship the authors present Eq. 12. The corresponding equation found by the writer (18) is

[a]June, 1982, by A. Mahinda Samarasinghe, Yang H. Huang, and Vincent P. Drnevich (Paper 17153).

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